



Modeling techniques for nonlinear site response;  
Developments, limitations and future directions

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***Where you can get a taste of the arctic without going to the North Pole***

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# Acknowledgments

- Jonathan Stewart
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- Neven Matasovic
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- National Science Foundation
- USGS

# Outline

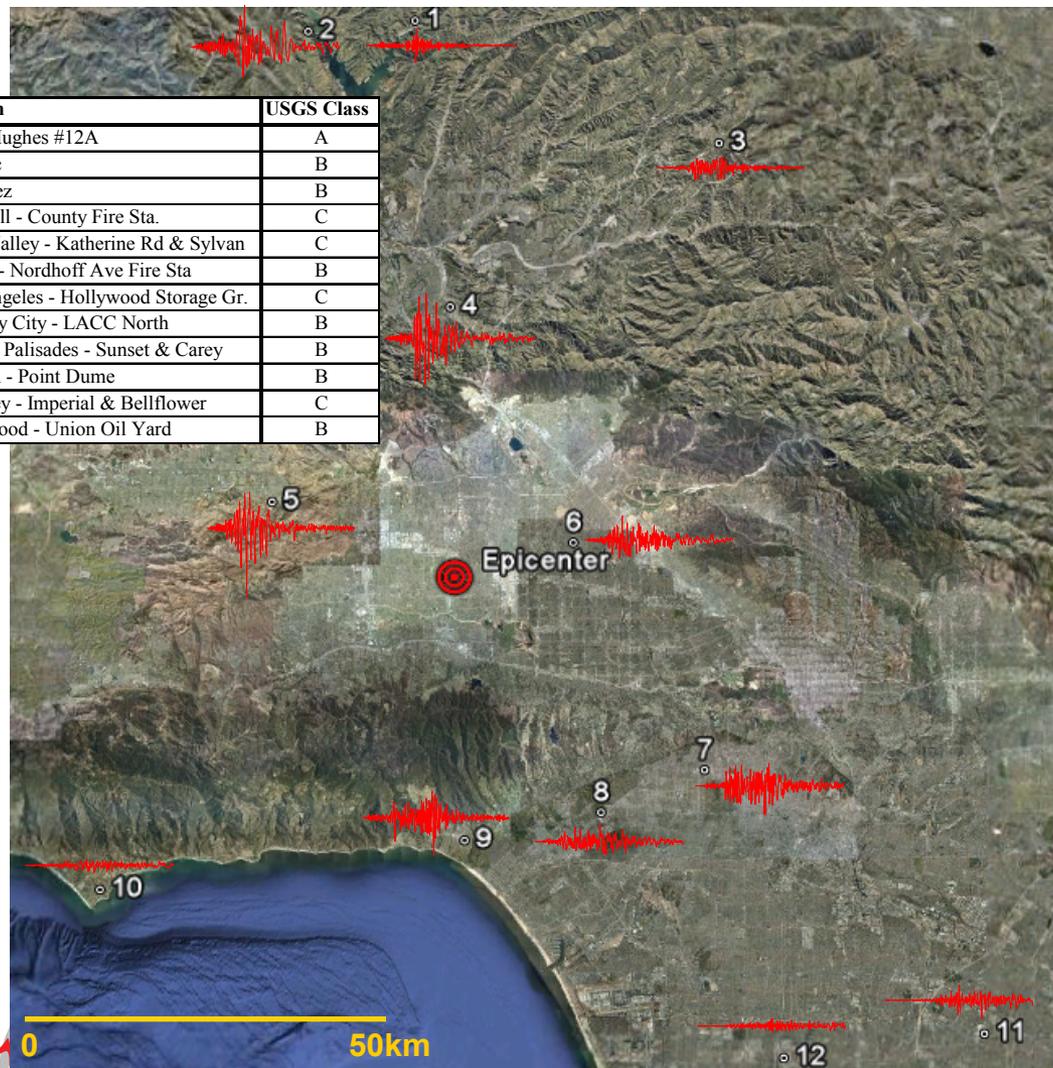
- Motivation and field evidence
- Analysis methods
  - Equivalent Linear
  - Nonlinear
- Solution of equation of motion & soil modeling:
- Soil properties and nonlinear curves
  - Viscous and hysteretic damping
  - Matching modulus reduction & damping curves
  - Implied shear strength
  - Porewater pressure generation
- Miscellaneous issues:
  - Layer thickness
  - Outcrop vs within motion
  - Input motion time step and response spectrum calculation
- Criteria for EL-NL selection
- Concluding Remarks

# Impacts – Site Effects

- Ground motions well recorded (157 recordings on six networks)
- Local damage correlated with site amplification in Sherman Oaks, Santa Monica, and west LA
- Nonlinear site effects

# Impacts - Field Evidence

Id	Station	USGS Class
1	Lake Hughes #12A	A
2	Castaic	B
3	Vasquez	B
4	Newhall - County Fire Sta.	C
5	Simi Valley - Katherine Rd & Sylvan	C
6	Arleta - Nordhoff Ave Fire Sta	B
7	Los Angeles - Hollywood Storage Gr.	C
8	Century City - LACC North	B
9	Pacific Palisades - Sunset & Carey	B
10	Malibu - Point Dume	B
11	Downey - Imperial & Bellflower	C
12	Inglewood - Union Oil Yard	B



*Collapse of sections of Interstate 5  
Arnesen Photography, 1994*



*Building collapse GEES, 1994*

# AMPLIFICATION OF GROUND MOTION

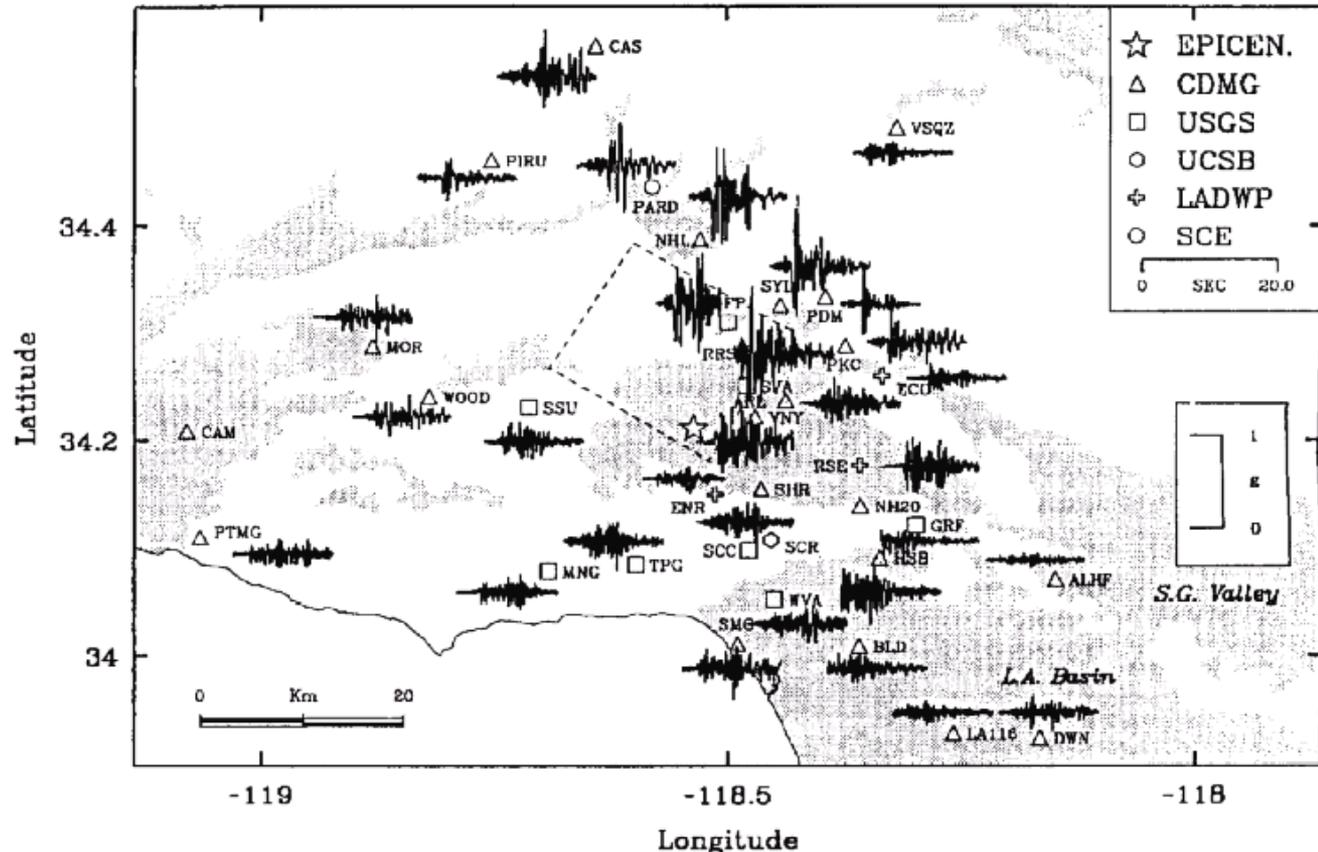
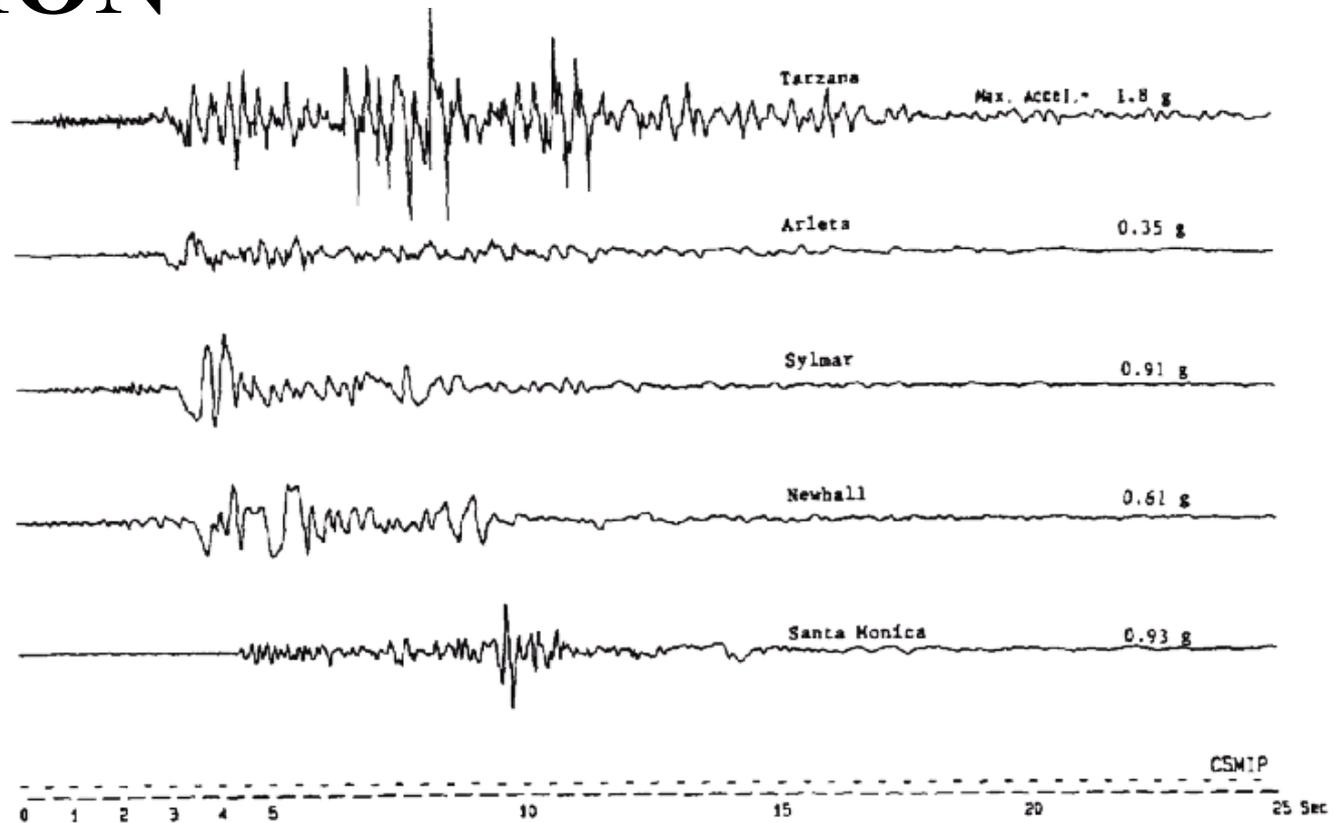


Figure 2-15a Distribution of ground motions for selected strong-motion stations: north component of acceleration. Time histories are plotted close to the associated site. Time and amplitude scales are shown to the right. Shaded areas represent alluvial basins and valleys.

From EQ Spectra

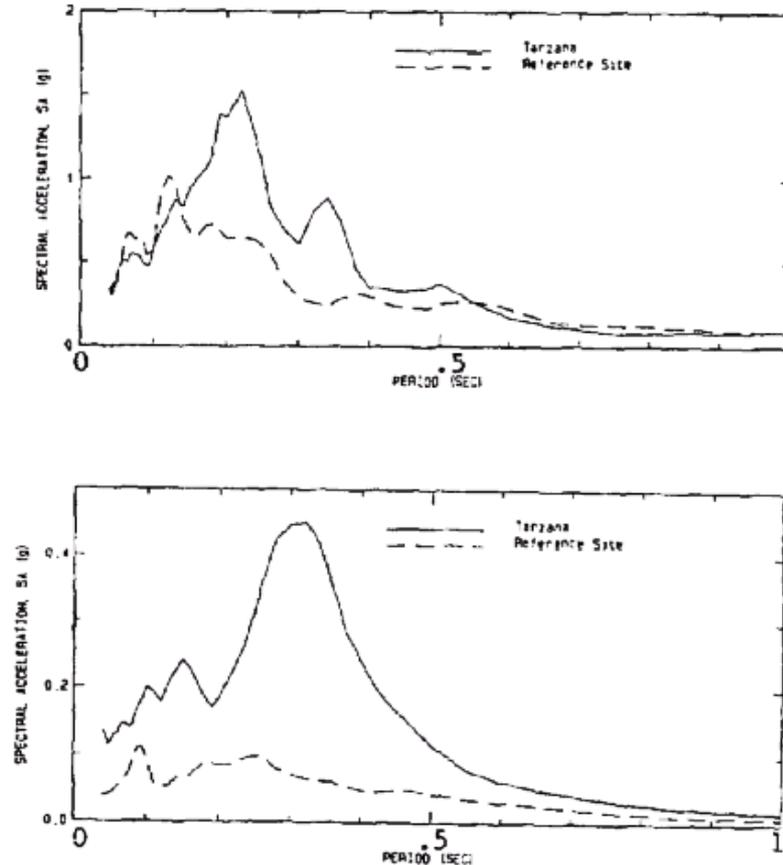


# AMPLIFICATION OF GROUND MOTION



**Figure 2-21** Comparison of acceleration waveforms at five ground-response stations within 25 km of the epicenter of the Northridge earthquake. Tarzana, Arleta, and Sylmar County Hospital are in the San Fernando Valley. Newhall is north of the Valley and Santa Monica is located to the south in the Los Angeles basin.

# Impacts – Northridge Site Effects



**Figure 2-23** Comparison of accelerograms and spectra (5% damped) for the two Northridge aftershock records from the Tarzana CSMIP station and a nearby reference site off the hill and about 120 meters from the Tarzana site. Peak accelerations of 0.26g at Tarzana and 0.25g at the reference site were recorded during the M5.3 aftershock on March 20, 1994. Peak accelerations of 0.12g at Tarzana and 0.04g at the reference site were recorded during the M4.4 aftershock of January 27.

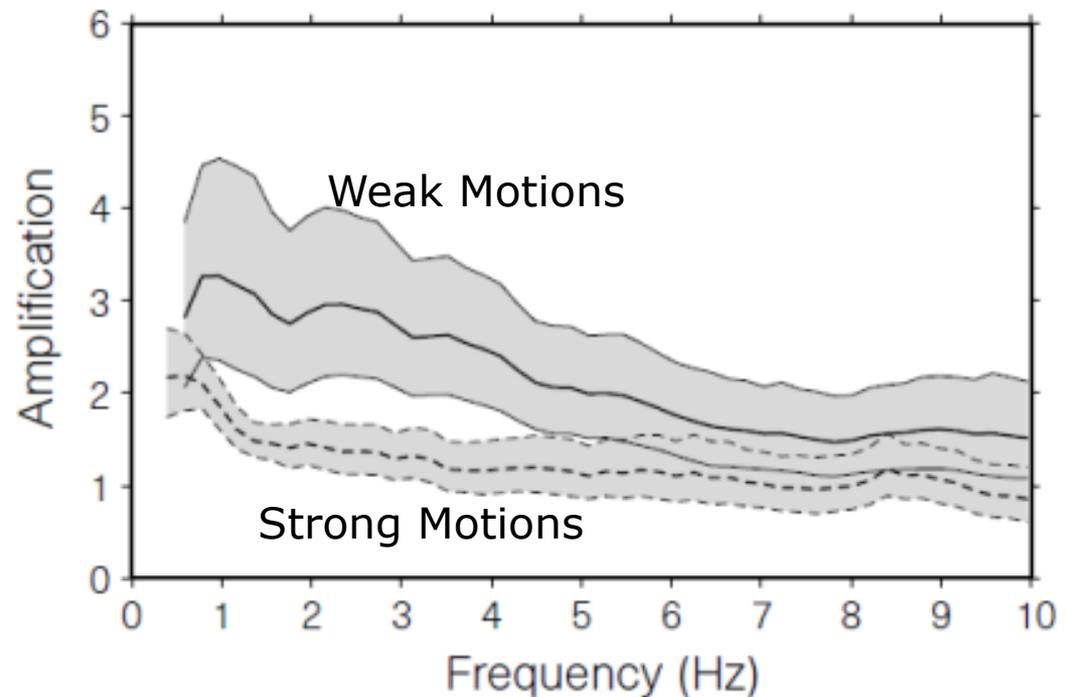
# Impact – Seismologists' recognition of non-linear site effects

Finally, the conclusion of significant nonlinearity is good news in that the amplifying effects of sediments, on average, are apparently not as great as implied by weak-motion studies. However, it brings into question the use of empirical Green's functions (based on recordings of small earthquakes) to study or predict strong ground motion at sediment sites.

## Nonlinear ground-motion amplification by sediments during the 1994 Northridge earthquake

Edward H. Field<sup>+</sup>, Paul A. Johnson<sup>†‡</sup>, Igor A. Beresnev<sup>§</sup> & Yuehua Zeng<sup>||</sup>

Nature – Dec 1997



# Impacts – Significance of local site response

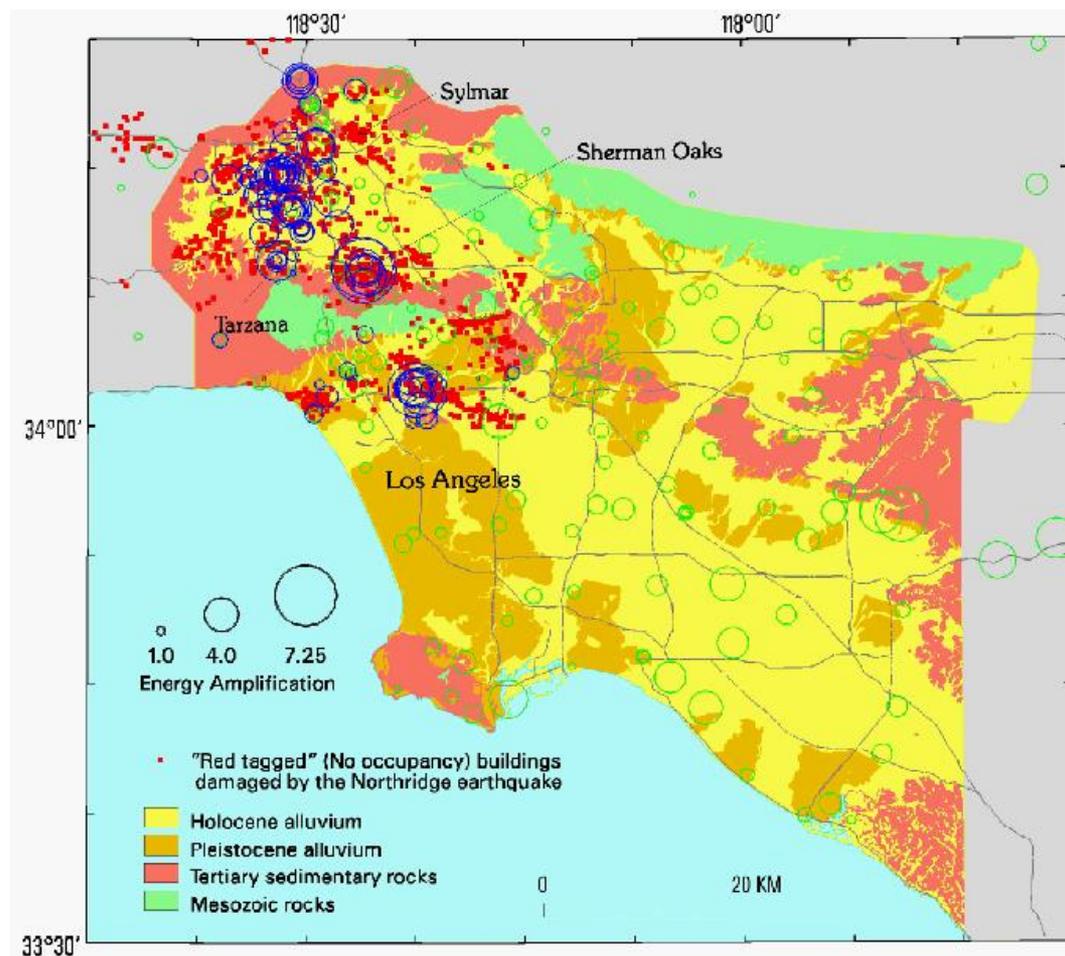
## ROSRINE:

(*ResO*lution of Site Response *I*ssues from the Northridge *E*arthquake) is a government-academia-industry research collaboration aimed at improving engineering models of earthquake ground motion through collection, synthesis, and dissemination of data on subsurface conditions at key Strong Motion (SM) station sites.

- Borehole data
- Geophysical data
- Laboratory testing

**Circles show energy amplification**

<http://rccg03.usc.edu/Rosrine/>



<http://greenwood.cr.usgs.gov/pub/open-file-reports/ofr-96-0263/p28study.jpg>

# Outcomes

- Extensive laboratory testing of dynamic response of soils (Darendelli and Menq Curves, Prof. K. Stokoe)
- Developments in 1-D nonlinear site response

# Site Response Analysis

▶ Frequency Domain (FD)

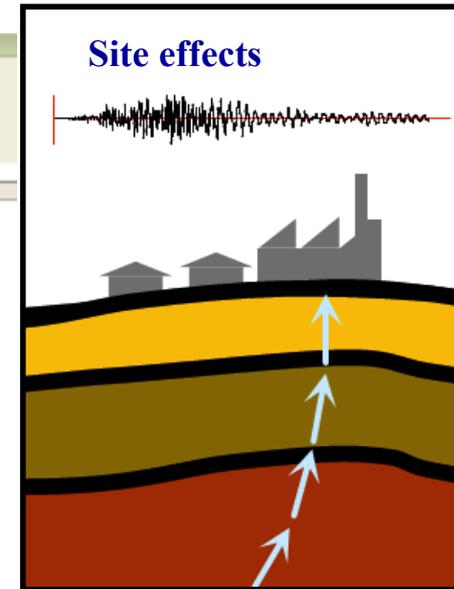
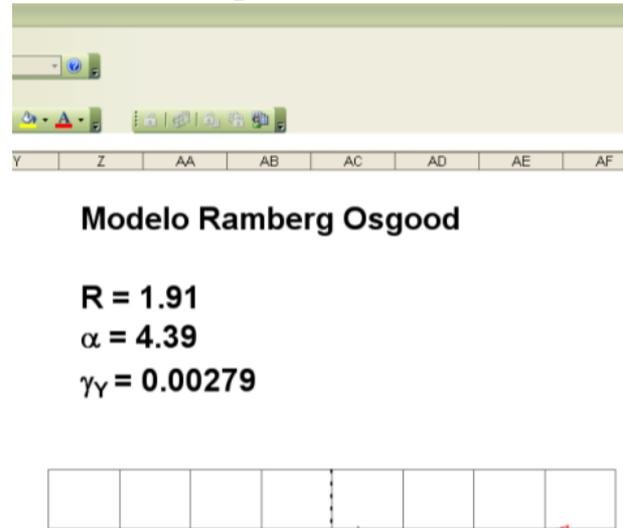
▶ Time Domain

Complexity of the problem:

▶ 1D

▶ 2D

▶ 3D



Dynamic Soil Properties

Wave propagation

## WAVE PROPAGATION IN THE GROUND

Fault

Path

Surficial layers



Source

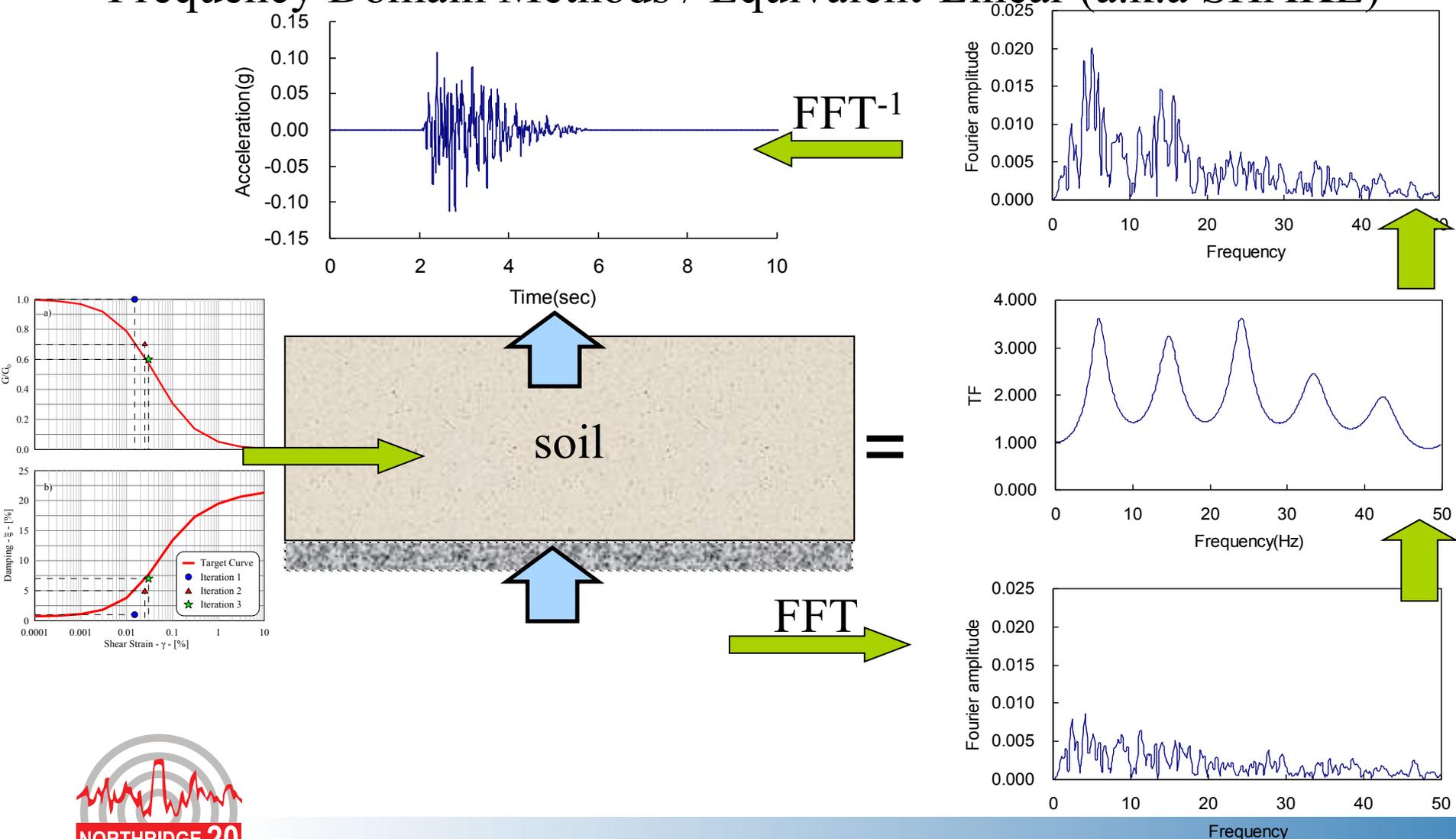
Seismic hazard assessment



1-D site response analysis for practical engineering applications.

# Site Response Analysis - EL

- Frequency Domain Methods / Equivalent-Linear (a.k.a SHAKE)



# Site Response analysis –EL

## ■ Advantages:

- Robust procedure
- Widely used
- Extensive evaluation

## ■ Issues

- Variation in stiffness with strain amplitude?
- Results under large strains or strong ground motion?
- Evaluation of pore water pressure generation?

# Site Response Analysis - NL

## 1D Wave Propagation – Time Domain Solution

Equation of Motion:

$$\underline{[M]} \{\ddot{u}\} + \underline{[C]} \{\dot{u}\} + \underline{[K]} \{u\} = -\underline{[M]} \{I\} \ddot{u}_g$$

$[M]$ : Massing matrix

→

Masses are typically calculated in each

time step for Damping

Exhibits frequency independent behavior

Frequency Independent Damping

(Osgood)

Modulus Reduction

Hysteretic Damping



# NL site response analysis - Barriers

Once upon a time...

- Inconsistent implementations
- Usage protocols
- Viscous damping
- Hysteretic damping when using Masing Rule
- Input motions
- Results that significantly vary from equivalent linear analysis
- Analysis time

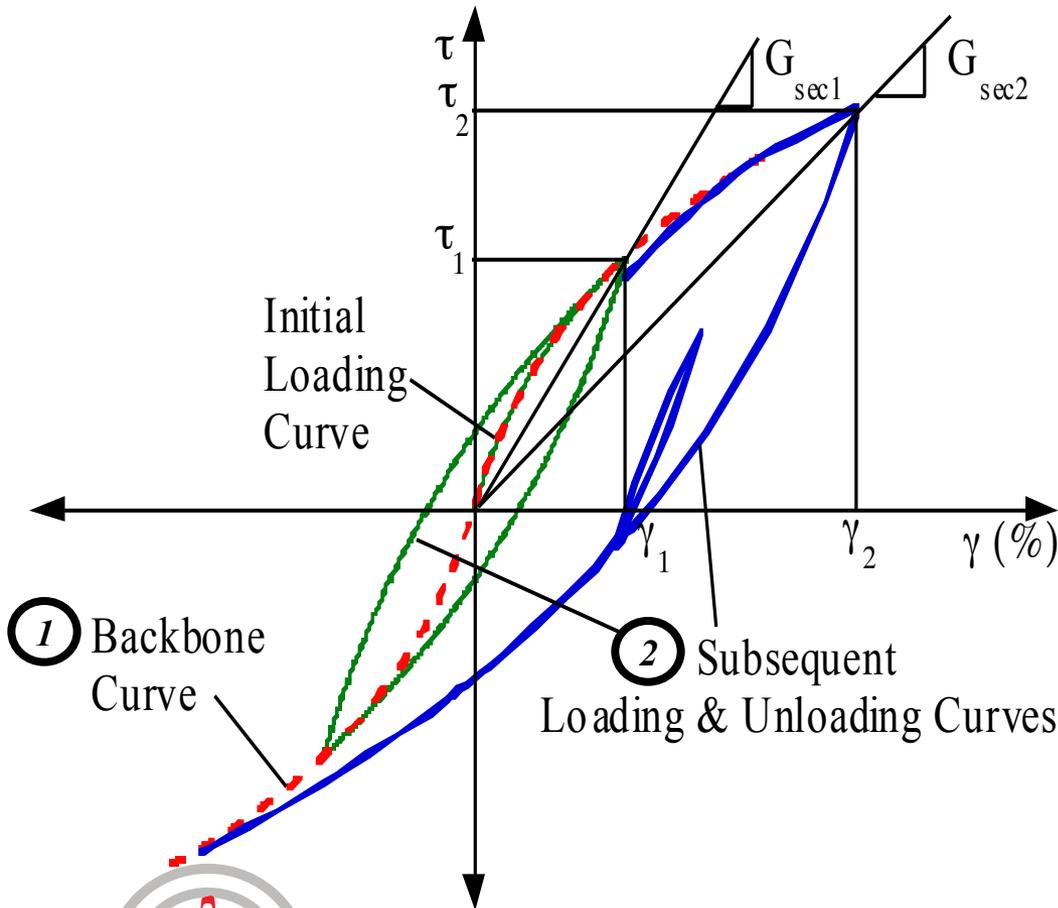
...greater user skill is required





# Simplified Shear Model for NL - SRA

## 1D Wave Propagation – Time Domain Solution



Modified Kondner-Zelasko (MKZ) model (Matasovic 1993)

$$① \quad \tau = \frac{\gamma \cdot G_0}{1 + \beta \left( \frac{\gamma}{\gamma_r} \right)^s}$$

$$② \quad \tau = \frac{2 \cdot G_0 \cdot \left( \frac{\gamma - \gamma_{rev}}{2} \right)}{1 + \beta \left( \frac{\gamma - \gamma_{rev}}{2 \cdot \gamma_r} \right)^s} + \tau_{rev}$$

Dynamic Equation :

$$[M] \ddot{u} + [C] \dot{u} + [K] u = -[M] \ddot{u}_g$$



# Viscous and Hysteretic Damping

Need to correctly represent damping (small & and large strains) in time domain non-linear analysis because:

- Small strain damping calculated using Rayleigh damping is frequency dependent → Inconsistent with available experimental data and current assumptions in damping curves.
- Use of extended Masing rules makes it difficult to represent simultaneously the observed changes of stiffness and energy dissipation (damping).
- Cumulative effects for softer or deeper soil profiles (e.g. New Madrid Seismic Zone, Sacramento River Delta) or large strain levels

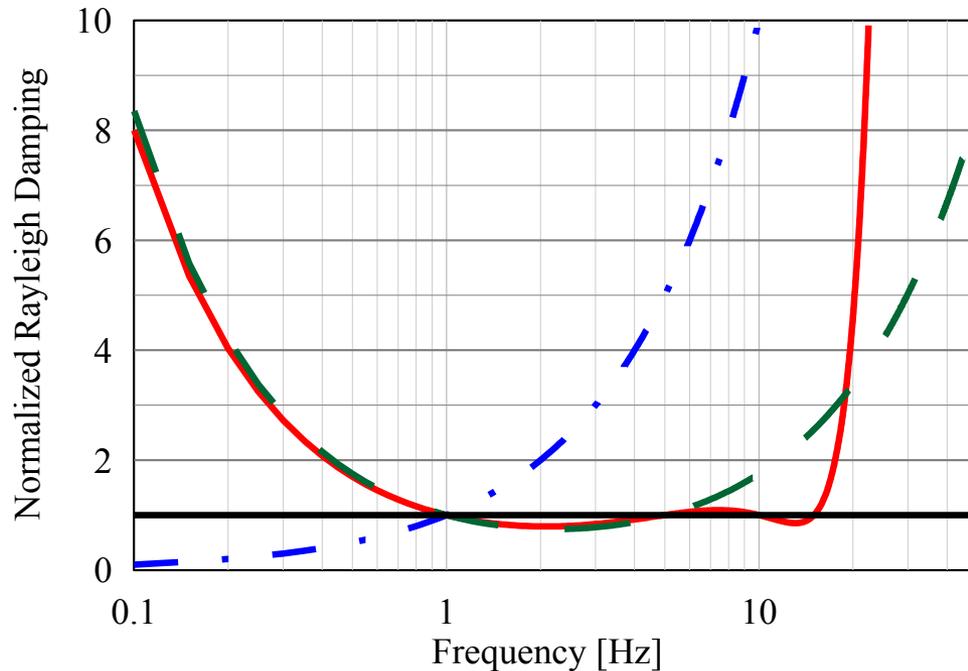


# Viscous Damping - Rayleigh Damping

Dynamic Equation :

$$[M] \ddot{u} + [C] \dot{u} + [K] u = -[M] \ddot{u}_g$$

$$[C] = [M] \sum_{b=0}^{N-1} a_b ([M]^{-1} [K])^b$$



1, 2 or more control frequencies/ modes can be included

Over-damped

Frequency Independent

Under-damped

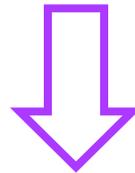
Number of Modes in Rayleigh Damping

— · — 1 Mode    - - - 2 Modes    — 4 Modes    — Target

# Frequency-Independent Viscous Damping

Formulation to construct the damping matrix

$$[C] = [M] \sum_{b=0}^{N-1} a_b \left( [M]^{-1} [K] \right)^b = [M] \sum_{b=0}^{N-1} a_b \Phi \omega^{2b} \Phi^{-1} = [M] \sum_{b=0}^{N-1} a_{1/2} \Phi \omega \Phi^{-1}$$



$$\xi_n = \frac{1}{4\pi f_n} \sum_{b=0}^{N-1} a_b (2\pi f_n)^{2b} = \frac{1}{4\pi f_n} [a_{1/2} (2\pi f_n)] = \frac{1}{2} a_{1/2}$$

$\rightarrow a_{1/2} = 2\xi_n$

**Small strain damping independent of the frequency  $\rightarrow$  experimental results**

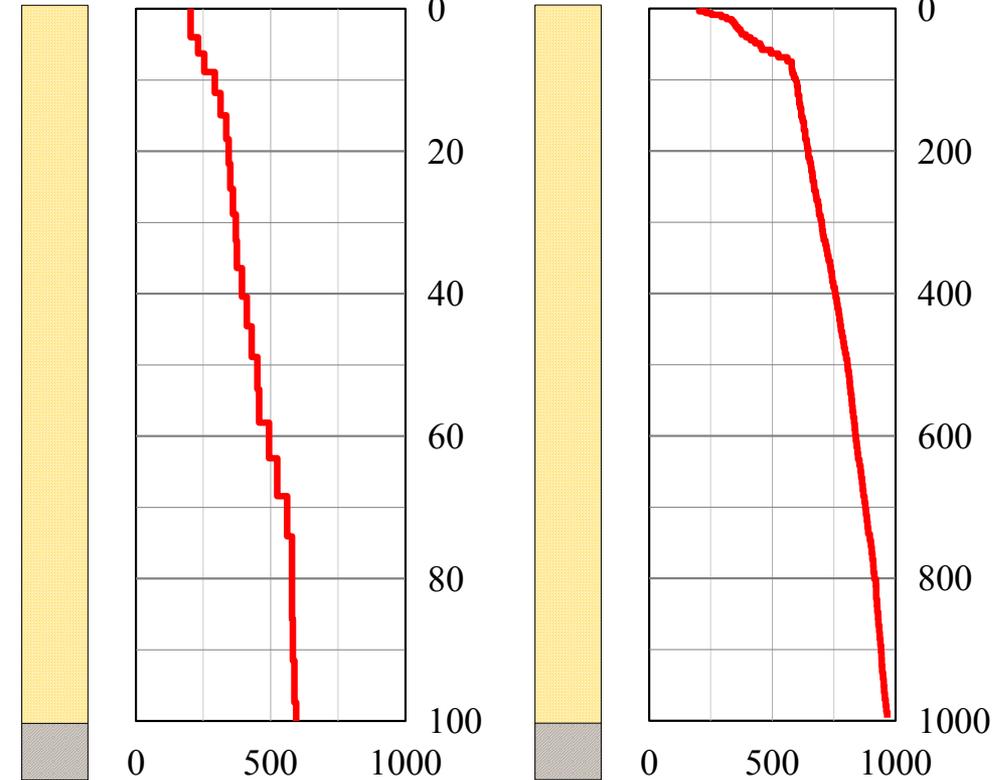
(Phillips and Hashash 2009, SDEE)

# Viscous Damping

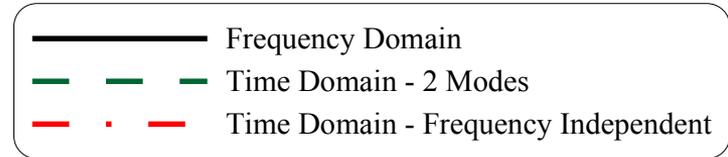
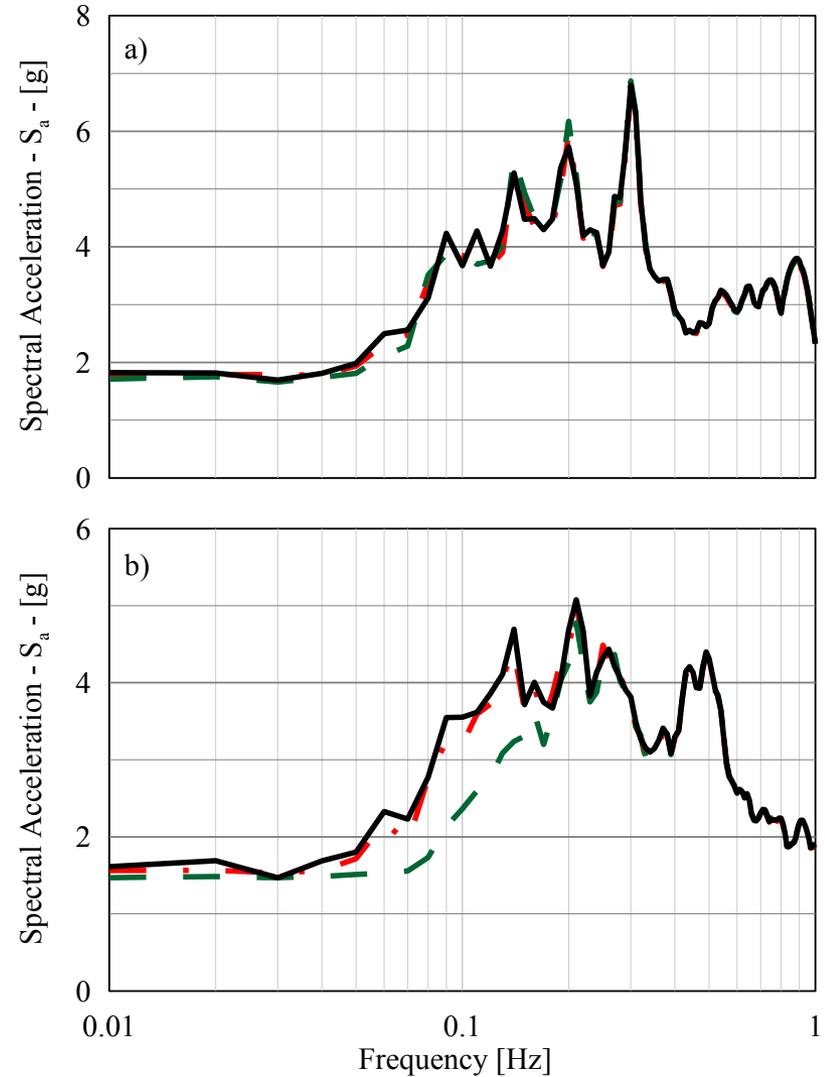
New Small Strain Damping

Example: Linear Elastic

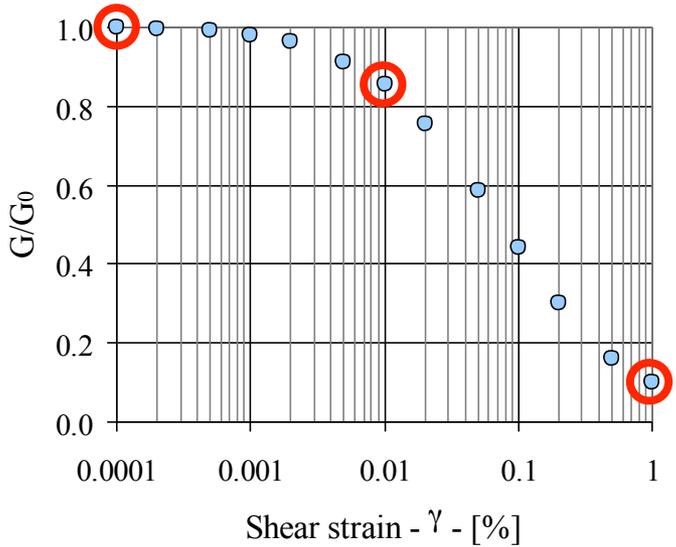
Frequency domain is exact solution



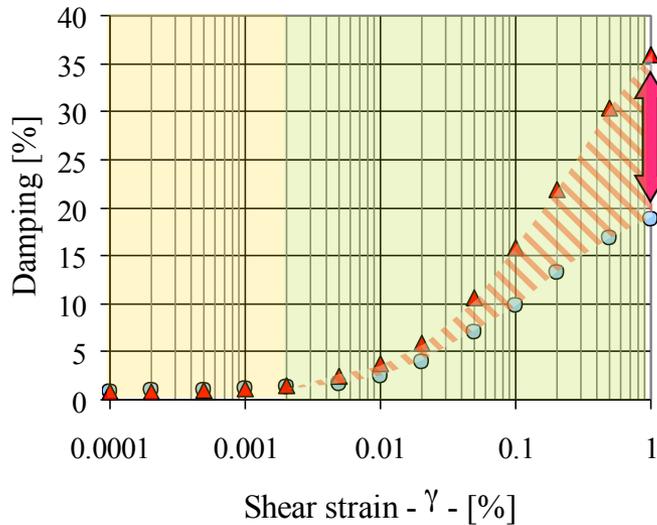
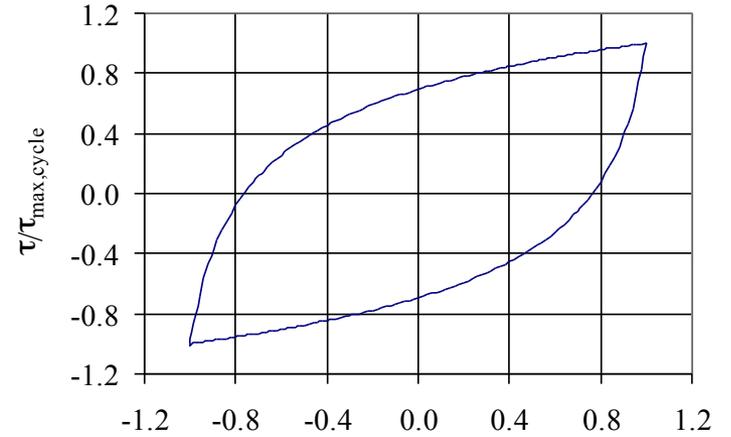
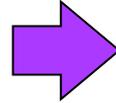
Shear Wave Velocity -  $V_s$  - [m/s]      Shear Wave Velocity -  $V_s$  - [m/s]



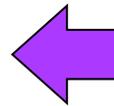
# Hysteretic Damping



Masing  
Rules

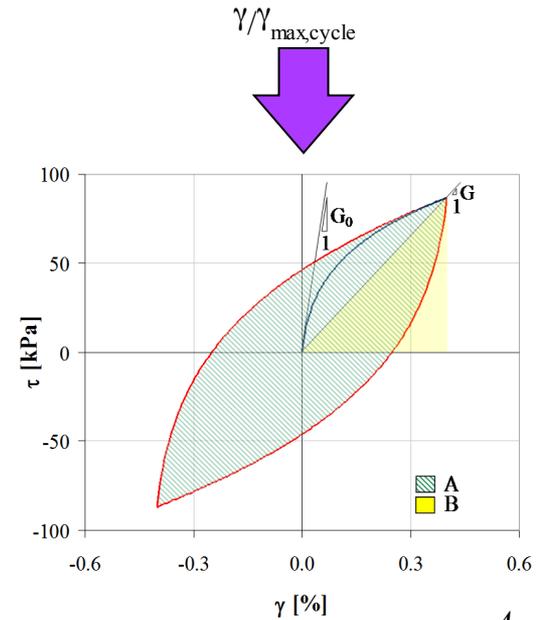


**Damping  
Mismatch**



Small strain Damping  
Hysteretic Damping

● Test Data    ▲ Calculated Value



$$\xi_{total} = \text{Small Strain Damping} + \frac{A}{4\pi \cdot B}$$

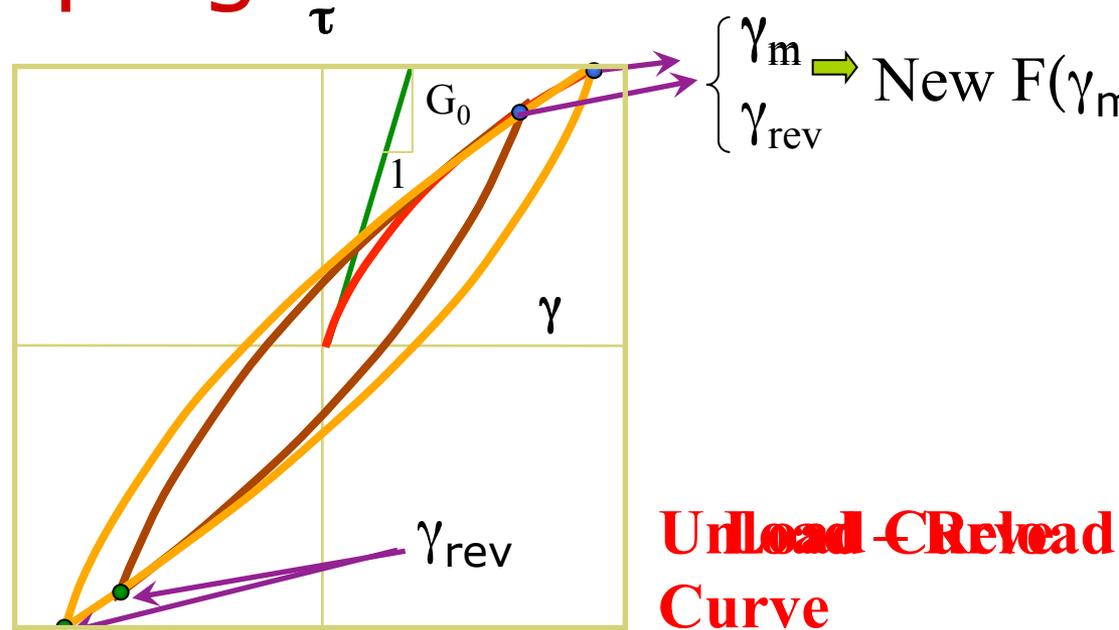
# Hysteretic Damping

Load Curve

$$\tau = \frac{G_0 \cdot \gamma}{1 + \beta \cdot \left( \frac{\gamma}{\gamma_r} \right)^s}$$

Unload – Reload Curve

adaptation of concept from Darendeli (non Masing rule)



$$\tau = F(\gamma_m) \cdot \left[ 2 \cdot \frac{G_0 \cdot \left( \frac{\gamma - \gamma_{rev}}{2} \right)}{1 + \beta \cdot \left( \frac{\gamma - \gamma_{rev}}{2 \cdot \gamma_r} \right)^s} - \frac{G_0 (\gamma - \gamma_{rev})}{1 + \beta \cdot \left( \frac{\gamma_m}{\gamma_r} \right)^s} \right] + \frac{G_0 (\gamma - \gamma_{rev})}{1 + \beta \cdot \left( \frac{\gamma_m}{\gamma_r} \right)^s} + \tau_{rev}$$

(Phillips and Hashash 2009, SDEE)

# Hysteretic Damping

## Non-Masing Rule Criteria

Use a Modulus Reduction Factor MRDF:

New Model

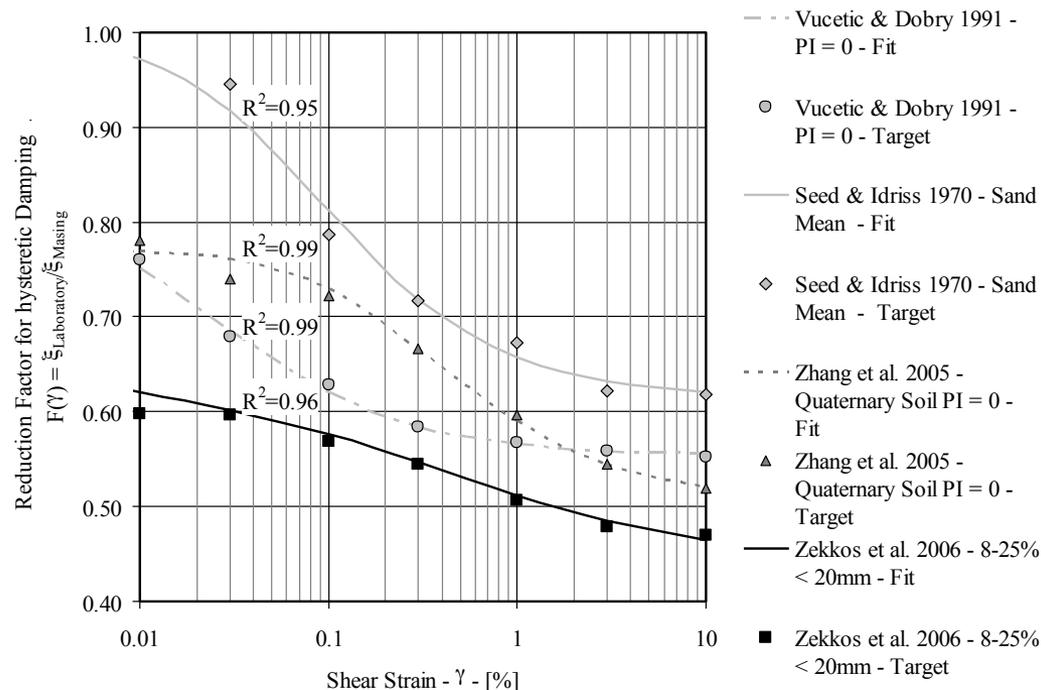
$$F(\gamma_m) = p_1 - p_2 \left( 1 - \frac{G_{\gamma_m}}{G_0} \right)^{p_3}$$



- Free Parameters

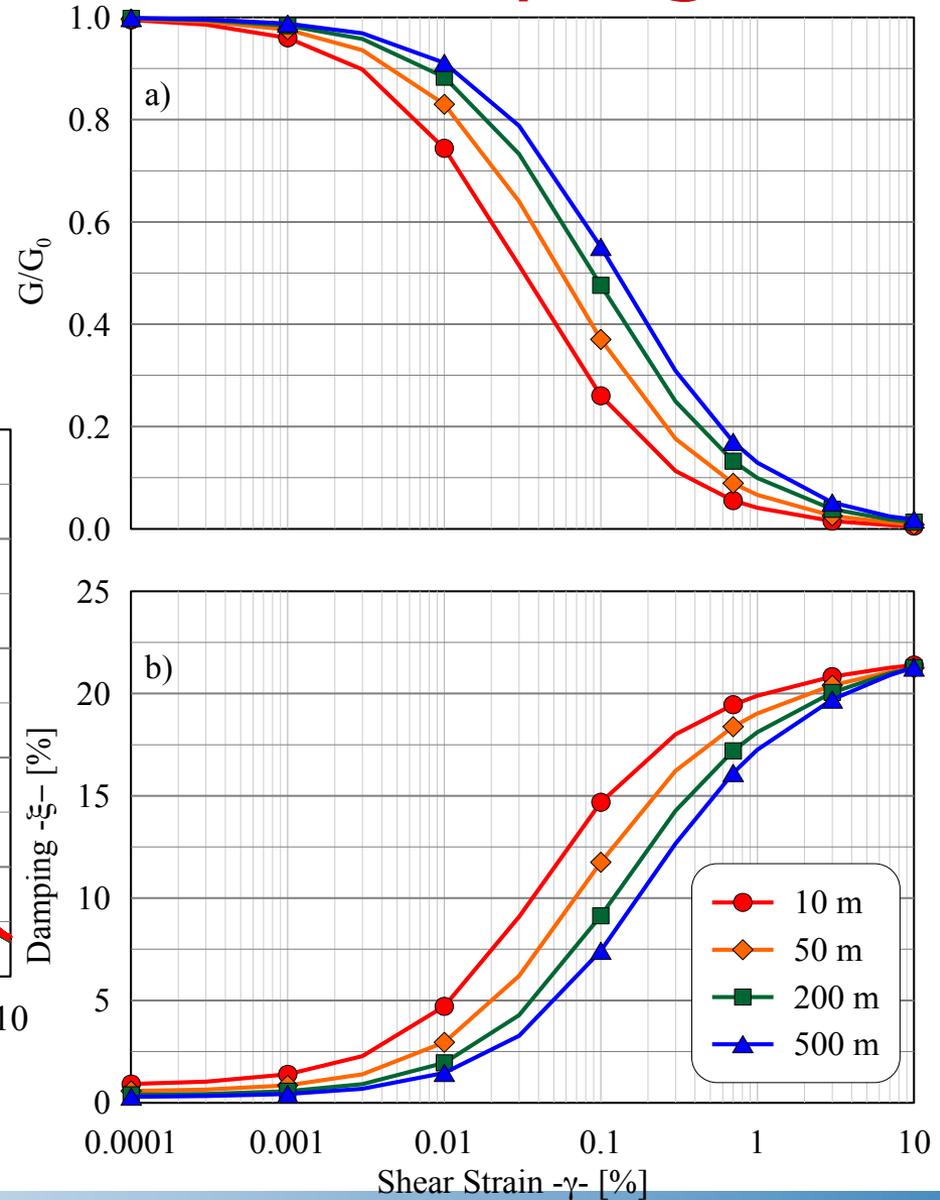
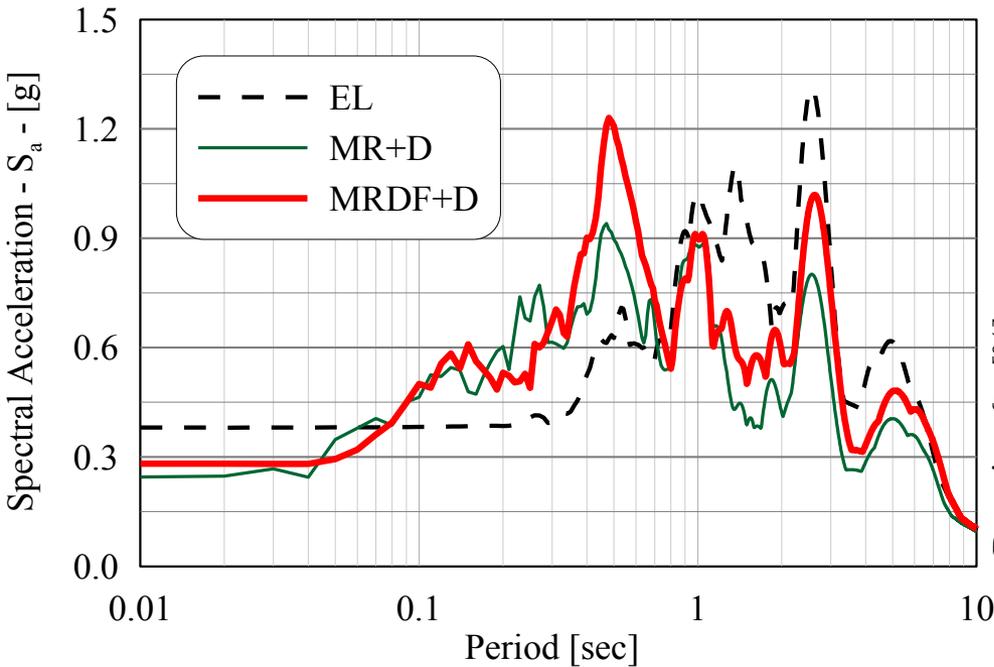
- Fitting Methodology

(Phillips and Hashash 2009, SDEE)



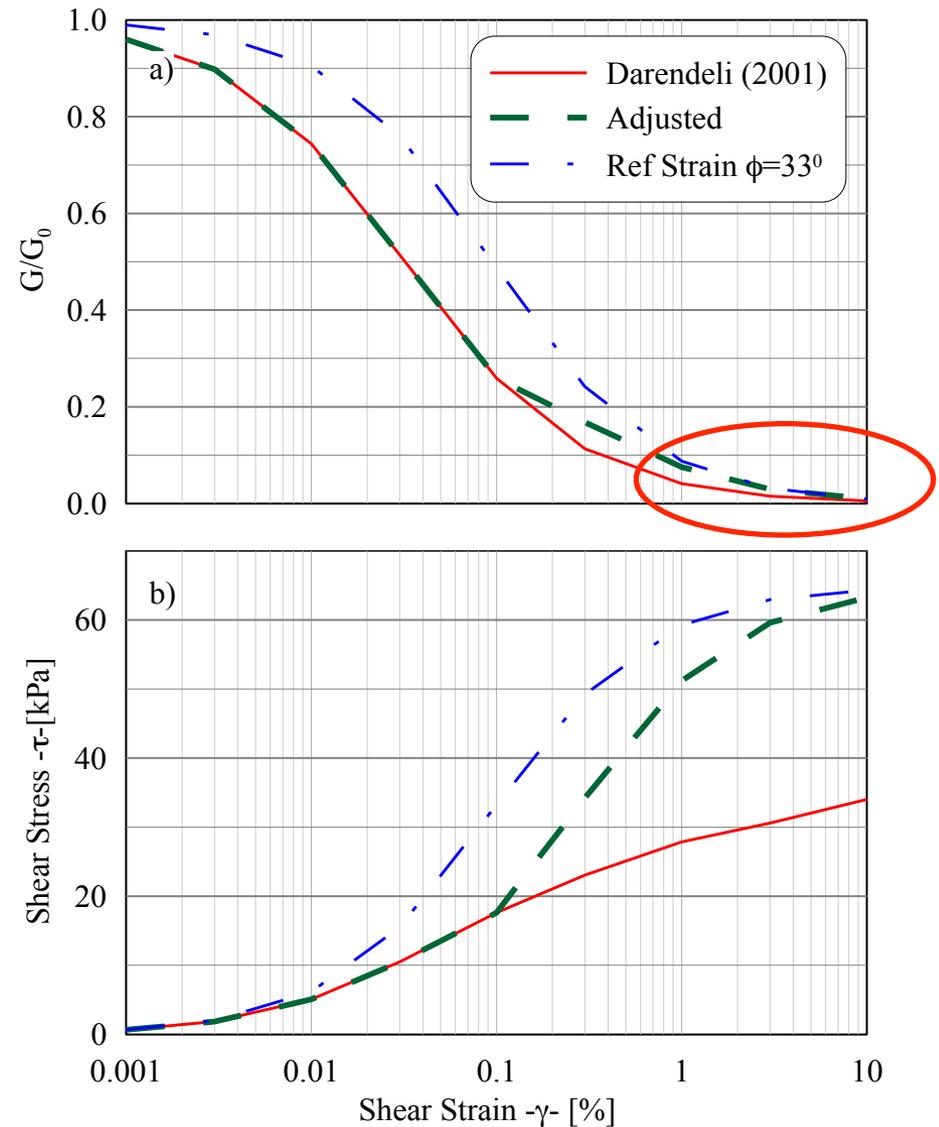
# Viscous and Hysteretic Damping

New Complete Model  
Non-Linear Example



# Implied Soil Strength At Large Strains

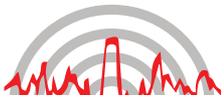
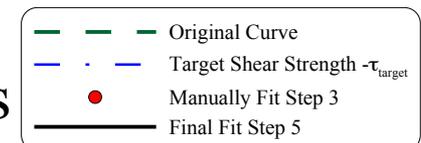
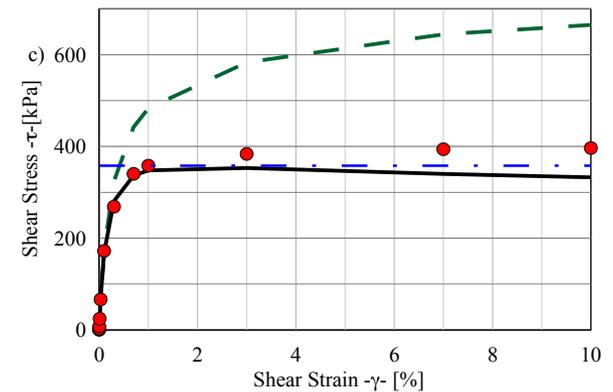
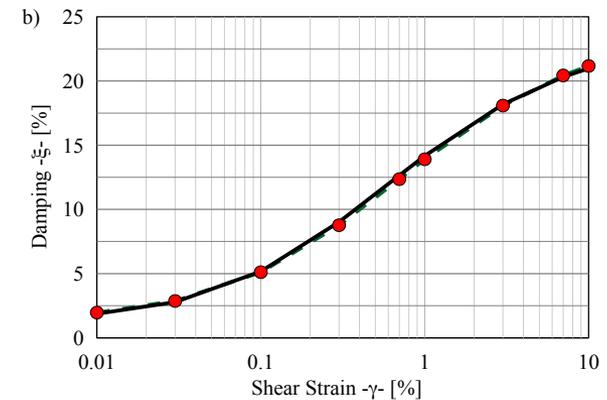
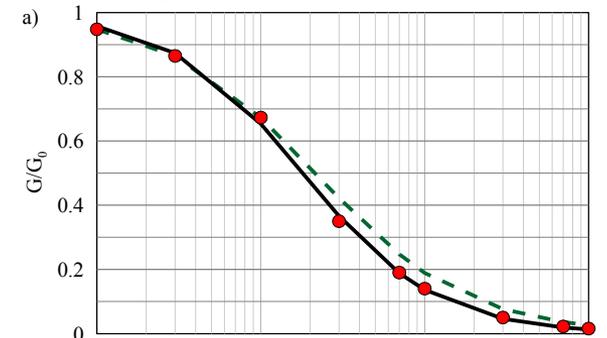
- Large strains in soft soils and due to strong shaking.
- Need for better resolution of implied strength or friction angle.
- Stewart and Kwok (2008)
- Suggested hybrid procedure for equivalent linear approach



# Implied Soil Strength At Large Strains

Iterative Procedure for NL backbone curve:

- 1) Fit the target using MRDF model.
- 2) Compute the implied soil shear strength
- 3) Underestimation: implied shear strength or friction angle is larger than the target value  
Overestimation: implied shear strength or friction angle is lower than the target value
- 4) Fit the modified modulus reduction curve (Step 3) and the damping curve obtained in Step 1 using the MRDF procedure.
- 5) Calculate the implied shear strength for the fitted curve using the aforementioned equations. If the implied shear strength is significantly higher or lower than the target value repeat Steps 3-5.



Need new functional forms and improved procedures

A new simple nonlinear model with  
input of soil strength

Under development in DEEPSOIL

# Overburden Pressure Dependent Properties

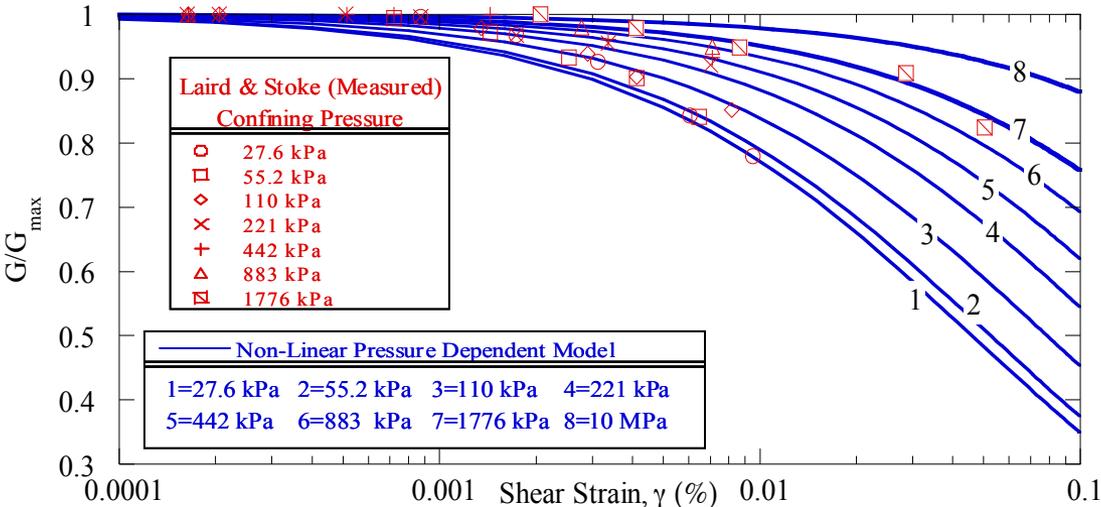
## G / G<sub>max</sub> & Confinement

### Reference strain

-confining pressure dependent

$$\tau = \frac{G_{mo}\gamma}{1 + \beta \left( \frac{G_{mo}}{\tau_{mo}} \gamma \right)^s} = \frac{G_{mo}\gamma}{1 + \beta \left( \frac{\gamma}{\gamma_r} \right)^s}$$

$$\gamma_r = a \left( \frac{\sigma'}{\sigma_{ref}} \right)^b$$



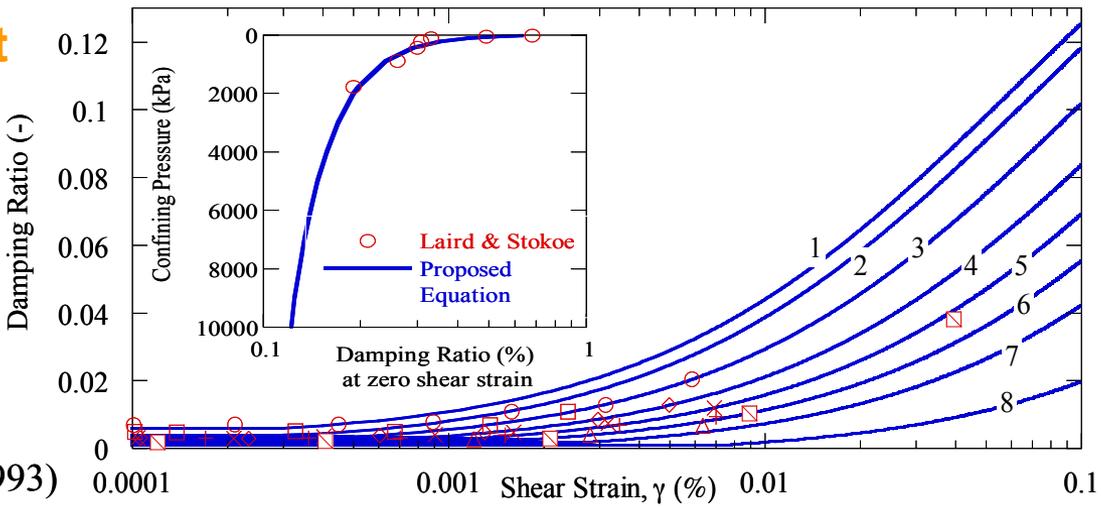
## Damping & Confinement

### Small strain damping

-Advanced formulation  
 -confining pressure dependent

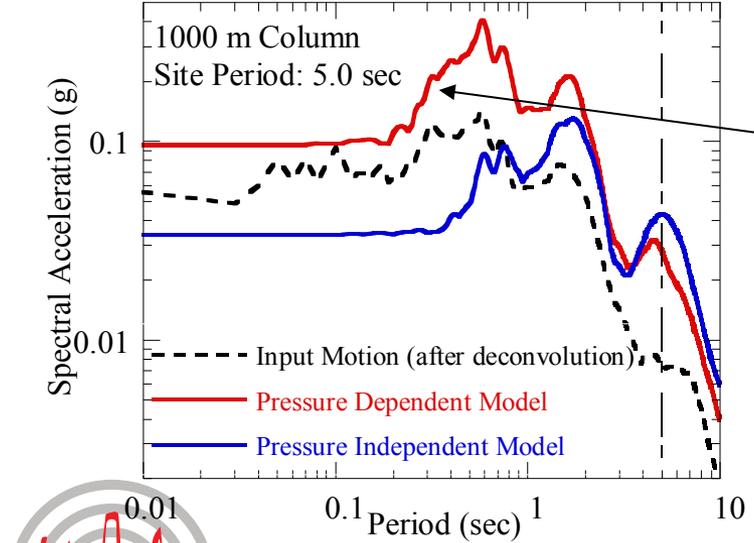
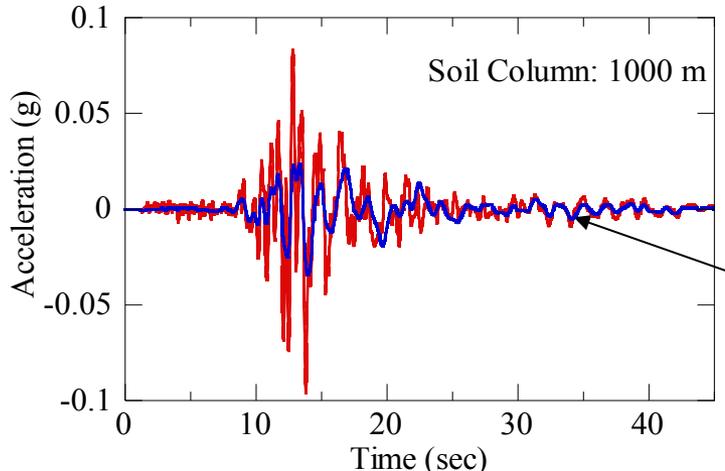
$$\xi = \frac{c}{(\sigma')^d}$$

Also EPRI (1993)

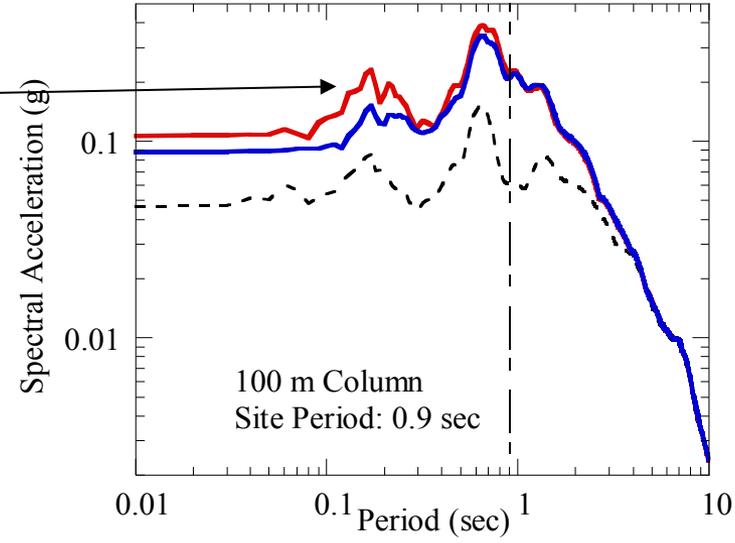
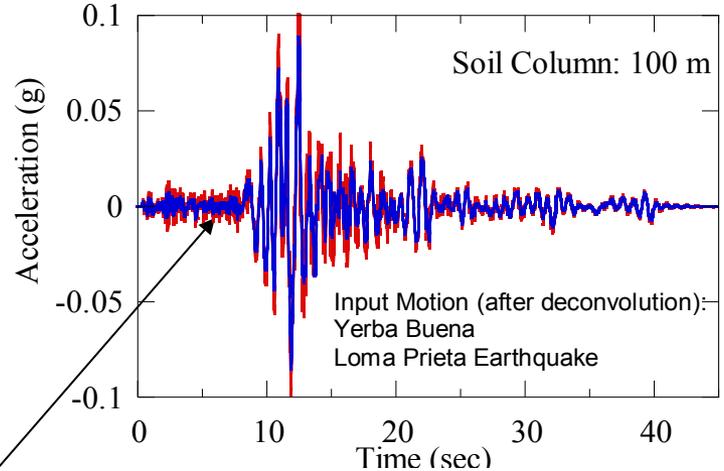


# Overburden Pressure Dependent Properties

1000 m Soil Column



100 m Soil Column



**Amplificati  
on  
of  
Long  
Period  
Waves**

**Propagati  
on  
of  
High  
Frequency  
Waves**



# Porewater Pressure Generation

## MRDF PWP Generation

- Generation of excess porewater pressures results in a reduction of soil stiffness, represented by a modulus degradation model and stress degradation model.

$$\delta = N \uparrow - t \quad \text{for} \quad \delta \downarrow G, \delta \downarrow \tau \quad (\text{Matasovic 1993})$$

$$t = s(\gamma \downarrow c - \gamma \downarrow tvp) \uparrow r$$

- Combine with Non-Masing Rule adaptation

$$\tau = F(\gamma \downarrow m) [2 \cdot G \downarrow 0 \cdot \delta \downarrow G (\gamma - \gamma \downarrow rev / 2) / 1 + \beta (\delta \downarrow G / \delta \downarrow \tau) \uparrow s (\gamma - \gamma \downarrow rev / 2 \cdot \gamma \downarrow r) \uparrow s - G \downarrow 0 \cdot \delta \downarrow G \cdot (\gamma - \gamma \downarrow rev) / 1 + \beta (\delta \downarrow G / \delta \downarrow \tau) \uparrow s (\gamma \downarrow m / \gamma \downarrow r) \uparrow s ] + G \cdot \delta \downarrow G \cdot (\gamma - \gamma \downarrow rev) / 1 + \beta (\delta \downarrow G / \delta \downarrow \tau) \uparrow s (\gamma \downarrow m / \gamma \downarrow r) \uparrow s + \tau \downarrow rev$$

# Pore Pressure Generation and Dissipation Models

## Stress-based pore pressure generation models

Seed et al. (1975) and Booker et al. (1976)

$$r_u = \frac{2}{\pi} \sin^{-1} \left[ \left( \frac{N}{N_{liq}} \right)^{\frac{1}{2\theta}} \right]$$

## Strain-based pore pressure generation models

Vucetic and Dobry. (1988)

$$u_N^* = \frac{p \cdot f \cdot F \cdot N \cdot (\gamma_c - \gamma_{tvp})^s}{1 + f \cdot F \cdot N \cdot (\gamma_c - \gamma_{tvp})}$$

$$G^* = G_0 \cdot \sqrt{1 - u_N^*}$$

$$\tau^* = \tau \cdot (1 - u_N^*)$$

# Porewater Pressure Dissipation

## GMP Energy-Based PWP Dissipation (Green et. al 2000)

- Relates generation of excess pore pressure to the energy dissipated per unit volume of soil

Energy dissipated per unit volume  $W_s$  and 'pseudo energy capacity' (PEC)

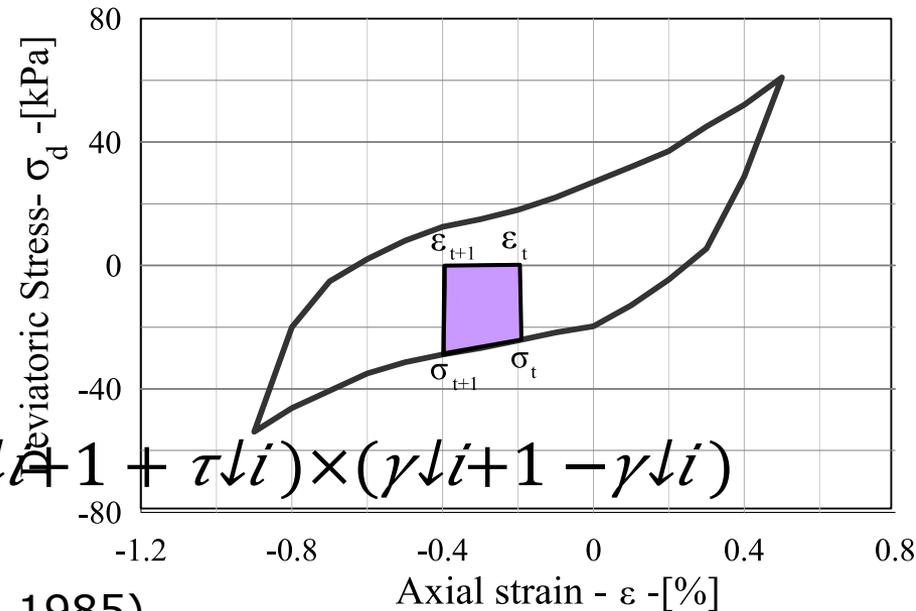
$$r \downarrow u = \sqrt{W \downarrow s} / PEC$$

$$W \downarrow s = 1/2 \sigma' \downarrow 0 \sum_{i=1}^{n-1} (\tau \downarrow i+1 + \tau \downarrow i) \times (\gamma \downarrow i+1 - \gamma \downarrow i)$$

(Berrill and Davis 1985)

$$\ln(PEC) = \begin{cases} \exp(c \downarrow 3 \cdot D \downarrow r) + c \downarrow 4 & FC < 35\% \\ c \downarrow 1 FC + c \downarrow 2 + \exp(c \downarrow 3 \cdot D \downarrow r) + c \downarrow 4 & FC \geq 35\% \end{cases}$$

$c_1 = -0.597, c_2 = 0.312,$   
 $c_3 = 0.0139, \text{ and } c_4 = -1.021$



(Polito et al. 2008)

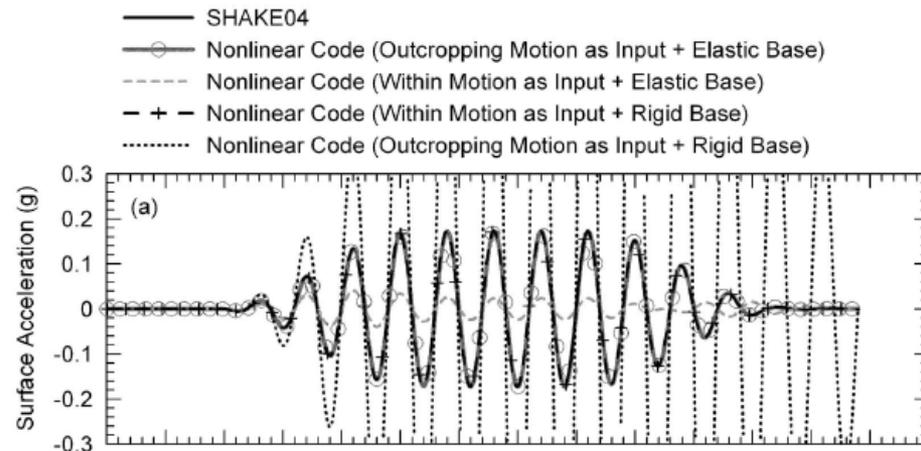
# Miscellaneous Considerations

- Maximum Layer Thickness  $f_{\max} = \frac{V_s}{4H_{\text{layer}}} \Rightarrow H_{\text{layer}(\max)} = \frac{V_s}{4f_{\max}}$

- Elastic vs Rigid Base  $\Leftrightarrow$  outcrop vs within motion  
(See Kwok et al. 2007)

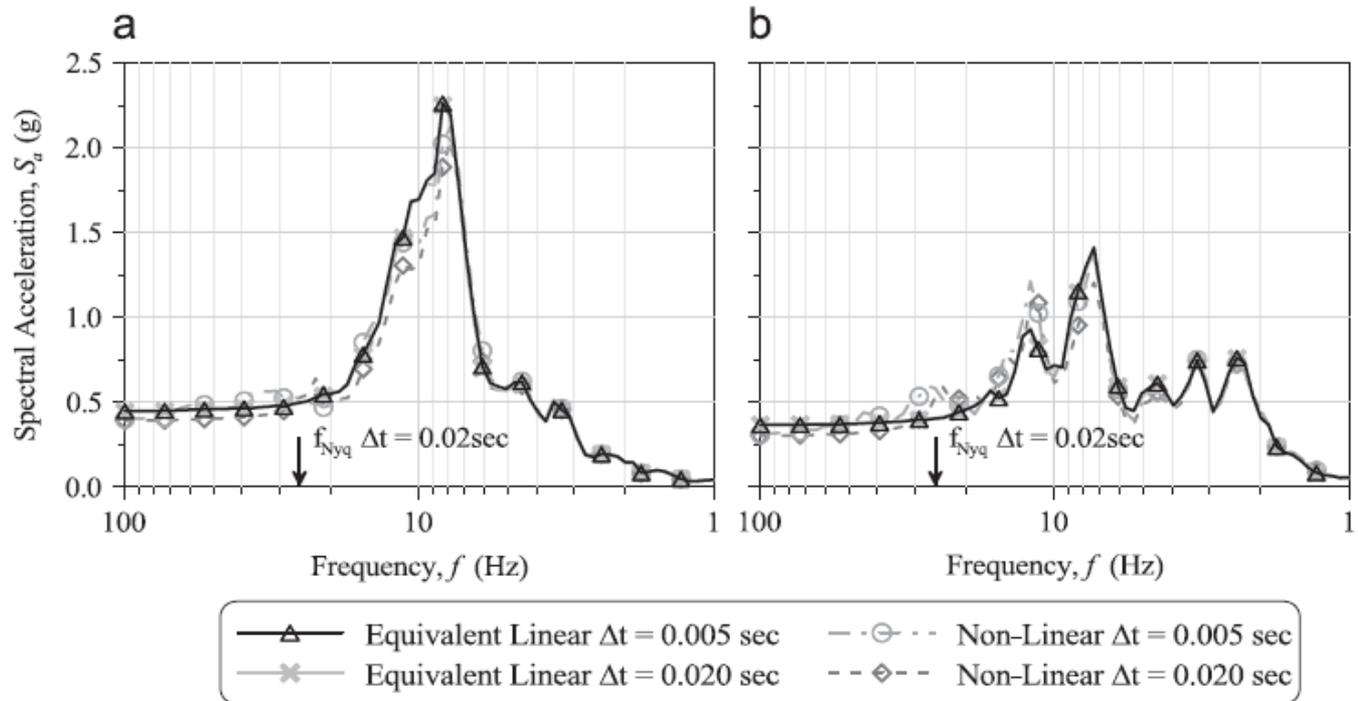
- **If only outcrop motion is available, we need to use elastic base:** since outcrop motion doesn't consider soil-rock interaction, we need to use elastic base to account for it.

- **If within motion is available, we need to use rigid base:** since within motion already considers soil-rock interaction (e.g. vertical array), we need to use rigid base to avoid accounting for soil rock interaction twice.



# Effect of input time step

- Nonlinear soil model with viscous and hysteretic damping



- EL (freq. domain) and NL (time domain) solutions are similar for  $\Delta t = 0.005$  sec
- Time step effects are important for time-domain analysis.

When is NL site response analysis needed?

Equivalent Linear vs Nonlinear

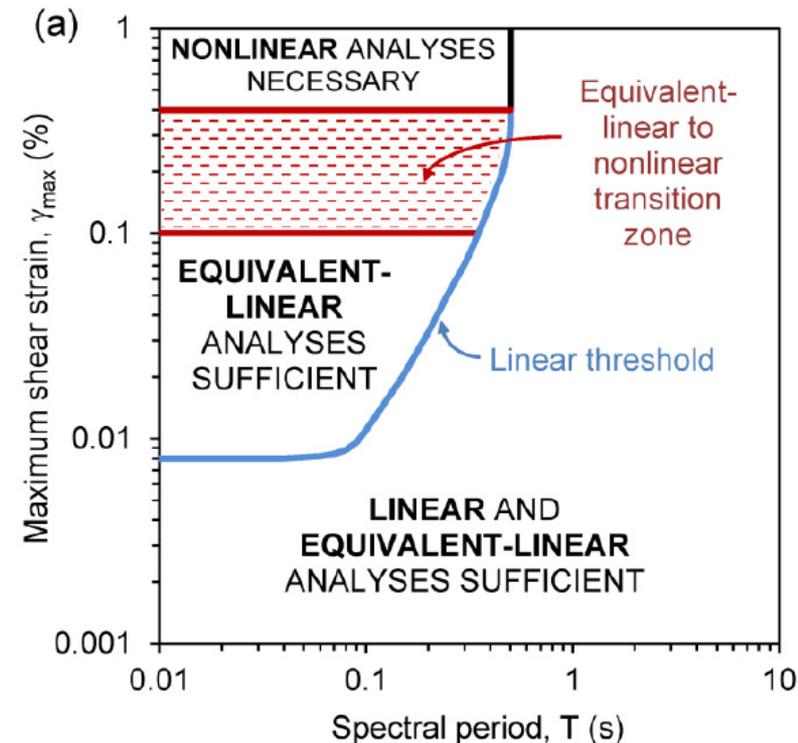
# Previous studies – NL vs. EL

- Matasovic and Hashash (2012)

- Survey: for some users NL when  $\gamma_{\max} > 1\%$

- Kaklamanos et al. (2013)

- 100 KiK-net downhole arrays in Japan.
- Compared recordings and estimated accelerations (by linear and equivalent-linear analyses)



$\gamma_{\max}$  computed from site response analysis.  
Both studies do not provide predictive tools.

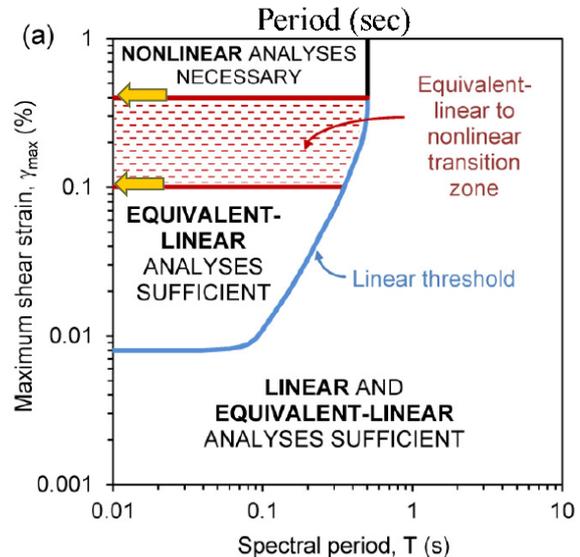
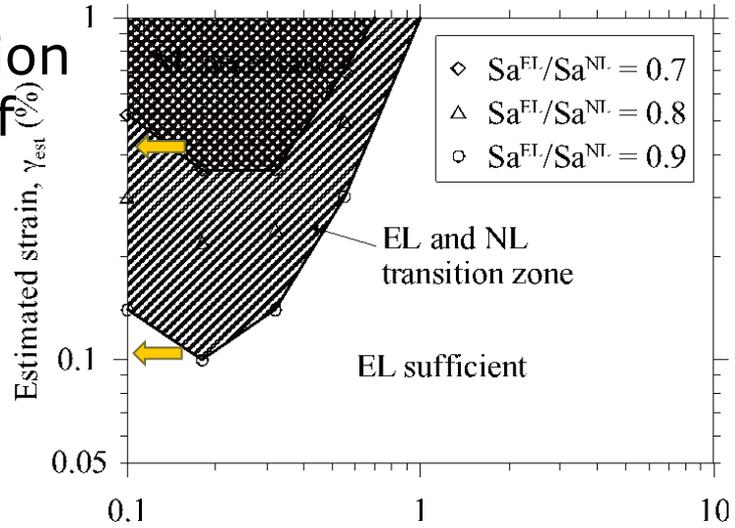
# Recommendations – est. strain: PGVin/Vs30

- Based on  $Sa^{EL}/Sa^{NL} = 0.7$  and  $0.9$ .
- Three regions (EL sufficient, transition zone, and NL necessary) in terms of  $\gamma_{est}$  and period.

EL sufficient – NL not needed.  
 Transition zone – equal weight for EL and NL.  
 NL necessary – greater weight for NL.

- Consistent with thresholds by Kaklamanos et al. (2013) – at 0.1 % and 0.4 %.

Correlation for different concepts for strains (estimated strain vs. maximum shear strain) needs to be addressed.



# User Interface for Robust 1d Site Response Analysis

## 1D Site Response Analysis (e.g. DEEPSOIL)

<http://www.illinois.edu/~deepsoil>

**Step 1/6: Choose Type of Analysis**  
To begin, either complete the fields in the "Create New Profile" section and select "Next", or press the "Open Existing Profile" button to open a saved profile.

**Create New Profile**

Layers  
# of Layers: 10

Units  
 English  Metric

Open Existing Profile...

**Analysis Method**

Frequency Domain:  
 Linear  Equivalent Linear

Time Domain:  
 Linear  Nonlinear

Analysis Type  
 Total Stress  Effective Stress  Include PWP Dissipation

Equivalent Linear  
Define Soil Curve by Using:  
 Discrete Points  Modified Hyperbolic Model

Nonlinear  
Define Soil Curve by Using:  
 Standard Hyperbolic Model  New Hyperbolic Model

Shear Input Properties By:  
 Modulus  Wave Velocity

Current Workspace Directory: C:\Program Files\UIUC\Deepsoil v3.5 BETA\Working\

Change Work Space... Next >>

**DEEPSOIL**

- Frequency Domain
  - Linear
  - Equivalent Linear
- Time Domain
  - Linear
  - Nonlinear
    - Total Stress
    - Effective Stress

# Concluding Remarks

- NL-SRA is now widely used
- Emphasis on simple soil models for ease of soil property selection
- Key developments :
  - small and large strain damping formulations,
  - pore water pressure generation models,
  - Mobilized shear strength
  - Increment of time steps
- Criteria for EL-NL selection is needed for effective simulation (Always perform EL).
- Carefully designed graphical user interfaces to improve quality of NL-SRA.

# Future Needs

- Implied strength
- Porewater pressure model calibration
- Uncertainty quantification

# THANK YOU

# QUESTIONS!

